Passbands and Stopbands for an Electromagnetic Waveguide with a Periodically Varying Cross Section

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Abstract —Electromagnetic waves in a rotationally symmetric and perfectly conducting waveguide with a periodically varying cross section are considered. Using the null field (T matrix) approach, a rather complicated determinantal condition for computing the axial wavenumber is derived. For a waveguide where the radius varies sinusoidally with the axial coordinate, the passbands and stopbands for the TE_{11}, TM_{11}, and TE_{12} modes are numerically computed. When the axial wavenumbers of two modes differ by a multiple of the wavenumber of the wall corrugations, the result is a stopband in the following cases: for two TE modes propagating in opposite directions, for a TE and a TM mode in the same direction, and sometimes for two TM modes in opposite directions.

I. Introduction

IN THE PRESENT PAPER, we consider the propagation of electromagnetic waves in a perfectly conducting rotationally symmetric waveguide whose wall has periodic corrugations that do not need to be small. This structure finds applications as a mode converter, for instance, and it has, therefore, been studied for small wall corrugations by Asfar and Nayfeh [1] and Kheifets [2] (further references can be found in these two papers). The main conclusion to be drawn from the literature is the appearance of resonances between two modes when the difference in wavenumber between the modes is equal to a multiple of the wavenumber for the wall corrugations. The resonance can be destructive, in which case it leads to a stopband. Some similar investigations of periodic structures—with similar results—include a rotationally symmetric acoustic duct [3], [4], a rectangular waveguide [5], and a parallel-plate waveguide [6]. For a review of waves in periodic structures in general, we refer to Elachi [7].

To perform our investigation, we employ the null field (or T matrix) approach (see [8]–[13] for some relevant applications). Especially useful for the present study is the paper by Boström [3] on acoustic waves in a cylindrical duct with periodically varying cross section and the calculation by Boström and Olsson [13] of the transmission and reflection by an obstacle inside a waveguide.

The main ideas of our approach are as follows. The starting point is a surface integral representation with the free-space Green's function. The Green's dyadic and the unknown surface field are then expanded in cylindrical vector waves, and by using the periodicity of the waveguide wall we obtain a determinantal condition for determining the axial wavenumber for the waveguide modes. The passbands and stopbands are then determined by whether this wavenumber is real or not. Numerical results are given for the modes corresponding to the TE_{11}, TM_{11}, and TE_{12} modes in a straight waveguide.

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II. Determination of the Waveguide Modes

Consider a cylindrical waveguide with a circular cross section and a wall S that is periodic in the axial z-direction. The equation of the wall is thus $\rho = \rho(z)$, where $\rho(z)$ is periodic with period $2a$. We assume time harmonic conditions, and the factor $\exp(-i\omega t)$ is suppressed. We take the medium in the waveguide to be homogeneous, isotropic, and lossless, so the electric field $\boldsymbol{E}$ satisfies

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - k^2 \boldsymbol{E}(\boldsymbol{r}) = 0 \tag{1}$$

where the wavenumber $k = \omega / c$ is real, c being the velocity of light. The waveguide wall is assumed to be perfectly conducting, i.e., the boundary condition is

$$\hat{n}' \times \boldsymbol{E}(\boldsymbol{r}') = 0, \qquad \boldsymbol{r}' \text{ on } S \tag{2}$$

where $\hat{n}'$ is the outward pointing unit normal on S. The problem we are addressing is the determination of the passbands and stopbands of the waveguide, or, phrased differently, the determination of the propagating modes (which are just the simplest type of solution of (1) and (2)).

To be systematic, we consider the field in the waveguide generated by some source, a dipole, for instance, inside the waveguide. Away from the source this field can then be written as a sum over the waveguide modes (propagating and nonpropagating). Thus solving this radiation problem we will, on the way, obtain the equations that determine the waveguide modes.

Our starting point is the following integral representation containing the free-space Green's function [13]

$$\boldsymbol{E}^i(\boldsymbol{r}) - k^{-2} \nabla \times \nabla \times \int_S G(\boldsymbol{r}, \boldsymbol{r}') \hat{n}' \times [\nabla' \times \boldsymbol{E}(\boldsymbol{r}')] \, dS' = \begin{cases} \boldsymbol{E}(\boldsymbol{r}), & \boldsymbol{r} \text{ inside } S \\ 0, & \boldsymbol{r} \text{ outside } S \end{cases} \tag{3}$$

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where the boundary condition (2) has already been employed and where $\boldsymbol{E}^i$ is the given field from the source. The free-space Green's function is

$$G(\boldsymbol{r},\boldsymbol{r}') = e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}/(4\pi|\boldsymbol{r}-\boldsymbol{r}'|) \tag{4}$$

and the expansion of the free-space Green's dyadic is [13]

$$\overleftrightarrow{I}G(\boldsymbol{r},\boldsymbol{r}') = i\sum_k \int_{-\infty}^{\infty} \mathrm{Re}\,\vec{\chi}_k(h;\boldsymbol{r}_<)\chi_k^\dagger(h;\boldsymbol{r}_>)\,dh + \overleftrightarrow{I}_{\mathrm{irr}} \tag{5}$$

where $\overleftrightarrow{I}$ is the unit dyadic and $\overleftrightarrow{I}_{\mathrm{irr}}$ is an irrotational dyadic. $\boldsymbol{r}_<(\boldsymbol{r}_>)$ denotes the radius vector with the smallest (greatest) value of ρ or ρ'. The cylindrical vector basis functions are here defined as

$$\chi_{\tau\sigma m}(h;\boldsymbol{r}) = (\epsilon_m/8\pi)^{1/2}(k/q)(k^{-1}\nabla x)^\tau \cdot\left[\hat{z}H_m^{(1)}(q\rho)\begin{pmatrix}\cos m\varphi\\ \sin m\varphi\end{pmatrix}e^{ihz}\right] \tag{6}$$

where $\epsilon_m = 2-\delta_{m0}$ (δ_{ml} is the Kronecker symbol), $q=(k^2-h^2)^{1/2}$, $\mathrm{Im}\,q \geq 0$ (Im stands for the imaginary part), and $H_m^{(1)}$ is the Hankel function of the first kind; $\tau = 1,2$ (TE and TM modes), $\sigma = e, o$ (determines azimuthal parity), and $m = 0,1,2,\ldots$. The regular basis functions contain a Bessel function J_m instead of $H_m^{(1)}$. The index k in the expansion (5) is a multi-index $k=(\tau\sigma m)$ and the dagger on χ_k (which can be moved to $\mathrm{Re}\,\chi_k$) means that e^{ihz} in (6) should be replaced by e^{-ihz} (before taking the curl).

Outside the circumscribed cylinder to S the field from the source can be expanded as

$$\boldsymbol{E}^i(\boldsymbol{r}) = \sum_k \int_{-\infty}^{\infty} a_k(h)\chi_k(h;\boldsymbol{r})\,dh/k \tag{7}$$

and inside the inscribed cylinder to S the scattered field $\boldsymbol{E}^s = \boldsymbol{E} - \boldsymbol{E}^i$ can be expanded as

$$\boldsymbol{E}^s(\boldsymbol{r}) = \sum_k \int_{-\infty}^{\infty} f_k(h)\,\mathrm{Re}\,\chi_k(h;\boldsymbol{r})\,dh/k. \tag{8}$$

Inserting the Green's dyadic (5) into the integral representation (3) and equating coefficients with (7) and (8) in their respective regions of validity yields

$$a_k(h) = ik\int_S \mathrm{Re}\,\chi_k^\dagger(h;\boldsymbol{r}')\cdot\hat{n}'\times[\nabla'\times\boldsymbol{E}(\boldsymbol{r}')]\,dS' \tag{9}$$

$$f_k(h) = -ik\int_S \chi_k^\dagger(h;\boldsymbol{r}')\cdot\hat{n}'\times[\nabla'\times\boldsymbol{E}(\boldsymbol{r}')]\,dS'. \tag{10}$$

To proceed we expand the surface field appearing in (9) and (10) in some suitable system

$$\hat{n}'\times[\nabla'\times\boldsymbol{E}(\boldsymbol{r}')] = \sum_{k'}\int_{-\infty}^{\infty}\alpha_{k'}(h')\zeta_{k'}(h';\boldsymbol{r}')\,dh', \quad \boldsymbol{r}' \text{ on } S. \tag{11}$$

Several different expansion systems are possible; we could, for instance, use the regular or outgoing basis functions. The simplest choice is probably to use the surface basis functions (analogous to using the spherical harmonics on a closed surface)

$$\zeta_k(h;\boldsymbol{r}) = \begin{cases} (\epsilon_m/8\pi)^{1/2}\begin{pmatrix}\cos m\varphi\\ \sin m\varphi\end{pmatrix}e^{ihz}\hat{\varphi}, & \tau = 1\\ (\epsilon_m/8\pi)^{1/2}\begin{pmatrix}\cos m\varphi\\ \sin m\varphi\end{pmatrix}e^{ihz}\hat{n}\times\hat{\varphi}, & \tau = 2.\end{cases} \tag{12}$$

Note that this system is only useful on a rotationally symmetric waveguide wall where $\hat{\varphi}$ is tangent to the surface. Another useful choice is

$$\zeta_k(h;\boldsymbol{r}) = k^{-1}\hat{n}\times[\nabla\times\mathrm{Re}\,\chi_k(h;\boldsymbol{r})]. \tag{13}$$

In the present case the expansion in (11) is then only valid strictly on the surface. However, if we had expanded only the scattered field on the surface in the system in (13), that expansion would in fact represent the scattered field in the whole waveguide, cf. Millar [14].

Introducing (11) into (9) and (10) gives

$$a_k(h) = i\sum_{k'}\int_{-\infty}^{\infty}\mathrm{Re}\,Q_{kk'}(h,h')\alpha_{k'}(h')\,dh'/k \tag{14}$$

$$f_k(h) = -i\sum_{k'}\int_{-\infty}^{\infty}Q_{kk'}(h,h')\alpha_{k'}(h')\,dh'/k \tag{15}$$

where

$$Q_{kk'}(h,h') = k^2\int_S \chi_k^\dagger(h;\boldsymbol{r}')\cdot\zeta_{k'}(h';\boldsymbol{r}')\,dS' \tag{16}$$

and $\mathrm{Re}\,Q_{kk'}$ contains $\mathrm{Re}\,\chi_k^\dagger$ instead of $\chi_k^\dagger$. Eliminating $\alpha_{k'}(h')$ between (14) and (15) and inserting the result for $f_k(h)$ into (8), we have thereby formally solved the radiation problem. To obtain the solution as a sum over the waveguide modes the integral in (8) must be closed (which is only possible away from the source). The poles of $f_k(h)$ will then determine the wavenumbers of the waveguide modes. A remaining branch line integral is expected to be cancelled by the direct field from the source, cf. the case treated by Boström and Olsson [13].

To obtain a more concrete characterization of the waveguide modes, we now use the rotational symmetry and the periodicity of the waveguide to perform the φ integration and reduce the z integration to one period in the surface integral in (16). The φ integration gives a decoupling into even ($\tau\sigma = 1o, 2e$) and odd ($\tau\sigma = 1e, 2o$) modes, and as these two kinds of modes have the same propagation characteristics we from now on only consider the even modes (we can then omit the σ index altogether as it is given implicitly by τ). The result after integrating (16) is then

$$Q_{\tau m,\tau' m'}(h,h') = \delta_{mm'}(\pi k^2/a)\sum_l \delta(h'-h-l\pi/a) \times\int_{-a}^{a}\boldsymbol{F}_{\tau m}(h;\rho(z),z)\cdot\boldsymbol{G}_{\tau' m'}(h';\rho(z),z)\rho(z)\,dz/n_\rho \tag{17}$$

where l is summed over all integers and n_ρ is the ρ component of $\hat{n}$. $\boldsymbol{F}_{\tau m}$ and $\boldsymbol{G}_{\tau m}$ are $\chi^\dagger_{\tau m}$ and $\zeta_{\tau m}$, respectively, with the φ dependence left out

$$\left(\sqrt{\epsilon_m/2\pi}\begin{pmatrix}\cos m\varphi\\ \sin m\varphi\end{pmatrix}\right)$$

is replaced by (1). We now introduce (17) into (14) and (15), change $h \to h + n\pi/a$, put $n' = n + l$, and sum over n' instead of over l, to finally obtain

$$a^{(m)}_{\tau n}(h) = i\sum_{\tau' n'} \operatorname{Re} Q^{(m)}_{\tau n, \tau' n'}(h)\,\alpha^{(m)}_{\tau' n'}(h) \tag{18}$$

$$f^{(m)}_{\tau n}(h) = -i\sum_{\tau' n'} Q^{(m)}_{\tau n, \tau' n'}(h)\,\alpha^{(m)}_{\tau' n'}(h) \tag{19}$$

where

$$Q^{(m)}_{\tau n, \tau' n'}(h) = (k\pi/a)\int_{-a}^{a} \boldsymbol{F}_{\tau m}(h + n\pi/a; \rho(z), z) \cdot \boldsymbol{G}_{\tau' m}(h + n'\pi/a; \rho(z), z)\,\rho(z)\,dz/n_\rho \tag{20}$$

and $a^{(m)}_{\tau n}(h) = a_{\tau m}(h + n\pi/a)$ and similarly for $\alpha^{(m)}_{\tau n}(h)$ and $f^{(m)}_{\tau n}(h)$. As discussed above, the poles of $f_{\tau m}(h)$ determine the propagation constants of the waveguide modes, and as $Q^{(m)}_{\tau n, \tau' n'}(h)$ has no poles (a fact which is evident from (20)), the poles must occur at those values of h where $\operatorname{Re} Q^{(m)}_{\tau n, \tau' n'}(h)$ is a singular matrix, i.e.,

$$\det \operatorname{Re} Q^{(m)}_{\tau n, \tau' n'}(h) = 0. \tag{21}$$

Thus this is the condition that determines the axial wavenumbers h of the waveguide modes (propagating or nonpropagating). The passbands and stopbands for a particular mode are then given by whether the corresponding h is real or not.

The value of the axial wavenumber h is not unique; $h + n\pi/a$, for any integer n, is evidently a solution if h is one. But if we demand that the value of h varies continuously as we deform from a straight waveguide to the periodic waveguide at hand, then h is unambiguously determined. It should be stressed, however, that it is perhaps a little misleading (but convenient) to call h "the axial wavenumber," as there really is no well-defined wavenumber or phase velocity in the axial direction.

If it should happen that the axial wavenumbers of two modes (which may propagate in different directions) differ by a multiple of π/a (which is the wavenumber of the wall corrugations), then we should expect that some sort of resonance occurs. That this is indeed so has been shown by Asfar and Nayfeh [1] for small wall perturbations, and will be further illuminated in the next section. It seems that more often than not the resonance leads to a stopband.

III. Numerical Results and Discussion

We now turn to a numerical investigation of the passbands and stopbands for a waveguide whose radius varies sinusoidally along the axial distance

$$\rho(z) = a + d\cos \pi z/a$$

where the axial period has been chosen equal to the mean diameter. We have investigated the case $m = 1$ and the frequency range $ka < 6$. This value of m has been chosen because the fundamental mode (the mode with the lowest cutoff) has $m = 1$, and also because the case $m = 0$ is less interesting in that the TE and TM modes do not couple in that case. For $ka < 6$ up to three modes are propagating, in conventional notation these are the TE_{11} (the fundamental mode), TM_{11}, and TE_{12} modes. We retain these names for the modes also when the waveguide has a periodic cross section and the modes no longer are transverse electric or magnetic.

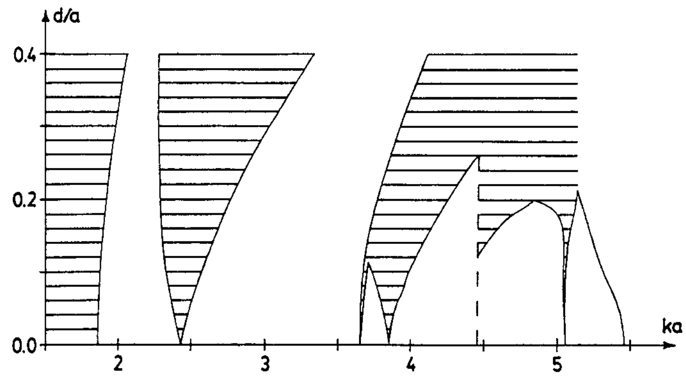

Fig. 1. The passbands and stopbands (shaded) for the TE_{11} mode.

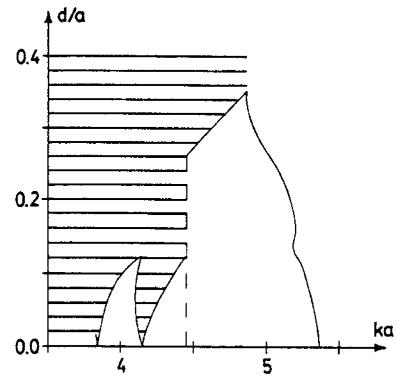

Fig. 2. The passbands and stopbands (shaded) for the TM_{11} mode.

When determining the passbands and stopbands numerically following the prescriptions of the previous section, we have usually employed the surface basis functions defined in (12). The regular basis functions, used according to (13), have only been employed as a check. At least for $d/a < 0.2$, the two alternatives give indistinguishable results, but when d/a is increased the regular basis functions become increasingly more difficult to use (cf. also the comments in the paper by Boström [3]). As a further check, we mention that the observation in the paper by Asfar and Nayfeh [1] that a wall perturbation decreases the wavenumbers of all TM modes not in resonance, is confirmed by our computations (no further comparisons with the work by Asfar and Nayfeh [1] are possible at present, because we have not considered two TM modes propagating in the same direction).

The passbands and stopbands of the TE_{11}, TM_{11}, and TE_{12} modes are shown in Figs. 1, 2, and 3, respectively. The frequency range is from cutoff to the last stopband for

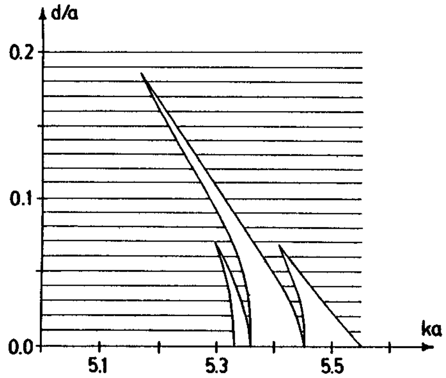

Fig. 3. The passbands and stopbands (shaded) for the TE_{12} mode.

TABLE I
THE CUTOFF AND RESONANCE FREQUENCIES, THE RESONATING MODES, AND THE TYPE OF THE RESONANCE FOR THE TE_{11}, TM_{11}, AND TE_{12} MODES IN THE FREQUENCY INTERVAL $ka < 6$ FOR A WAVEGUIDE WITH THE PERIOD OF THE WALL CORRUGATIONS EQUAL TO THE DIAMETER

frequency ka	modes		directions	type
1.841	TE_{11}	-	-	cutoff
2.420	TE_{11}	TE_{11}	opposite	stopband
3.641	TE_{11}	TE_{11}	opposite	stopband
3.832	TM_{11}	-	-	cutoff
3.838	TE_{11}	TM_{11}	same	stopband
4.141	TM_{11}	TM_{11}	opposite	stopband
4.440	TE_{11}	TM_{11}	opposite	passband
4.955	TM_{11}	TM_{11}	opposite	passband
5.059	TE_{11}	TE_{11}	opposite	stopband
5.331	TE_{12}	-	-	cutoff
5.367	TM_{11}	TE_{12}	same	stopband
5.454	TE_{11}	TE_{12}	opposite	stopband
5.558	TE_{12}	TE_{12}	opposite	stopband
5.622	TE_{11}	TM_{11}	opposite	passband
5.711	TM_{11}	TE_{12}	opposite	passband
5.852	TE_{11}	TE_{12}	same	passband

$ka < 6$, and the height of the corrugations are $0 \leq d/a \leq 0.4$. Together with the cutoffs, the resonances are also listed in Table I, which gives the resonance frequency, the resonating modes, the relative directions of the resonating modes, and the type of the resonance (stopband or passband). Our results may be summarized as follows: TE modes in opposite directions lead to a stopband and in the same direction they lead to a passband (in this respect TE modes behave as acoustic modes in a hard-walled waveguide [3]), a TE and a TM mode in opposite directions lead to a passband and in the same direction to a stopband, and TM modes in opposite directions can lead to either a passband or a stopband (we have not considered two TM modes in the same direction, but according to the paper by Asfar and Nayfeh [1] they should lead to a passband). These results are possibly true in general, although it should be noted that the only proved results are those listed in Table I. The passband at $ka = 4.955$, for the two TM_{11} modes propagating in different directions, is surprising—a stopband at this frequency and our results would be much more systematic and symmetrical. We have, however, made a thorough search (stepping in ka with steps of length 0.001 at several values of d/a), so it seems improbable that we should have missed a very narrow stopband.

The resonance between the TE_{11} and TM_{11} modes at $ka = 4.440$ is marked with a dashed line in Figs. 1 and 2. The boundaries between the stopbands and passbands are seen to jump at the resonance. The reason for this is that the two modes "cross over" at the resonance, i.e., what is the TM_{11} mode below $ka = 4.440$ becomes, upon continuous changes in ka with d/a fixed, the TE_{11} mode above. It should also be noted that around the resonance it is really meaningless to distinguish between the TE_{11} and TM_{11} modes, as the two resonance modes are a mixture of TE_{11} and TM_{11} (even for small d/a). The resonance at $ka = 4.955$ between the two TM_{11} modes in opposite directions also leads to a cross over, but it does not lead to a jump in the boundaries between the stopbands and passbands since the two modes have identical properties. The three resonances in the interval $5.6 < ka < 5.9$ are not included in Figs. 1–3 because we have not completely mapped the rather complex behavior of the three modes in this interval.

At $d/a = 0.4$, which is the highest value of d/a shown in the figures, there still exist some passbands. As in the acoustic case, cf. the paper by Boström [3], it is tempting to assume that some passbands may exist for all d/a up to $d/a = 1$, where they would go over into the sharp resonance frequencies of the resulting "onion-shaped" bounded volume.

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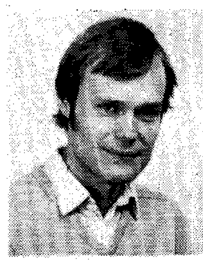

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Microwave Automatic Impedance Measuring Schemes Using Three Fixed Probes

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Abstract —**Following a previous article reporting a general theory and a new approach using multiple probes to measure the complex impedance of an unknown microwave load, this article describes a simplified, but improved, design derived from that general theory. A simple analog dc signal processor was built according to this design and preliminary experiments were carried out to check the performance of the system. Real time oscilloscope displays showing the complex reflection coefficients of some standard loads and some time-varying loads were recorded. The performance of this system was checked against that of the standard traveling probe technique. The maximum disagreement between the two methods is about 5 percent in amplitude and 7° in phase. A special dc signal processor—the display rotator—was used in the system. The purpose, the design, and the performance of this rotator circuit are discussed in detail. Although the present experiments are restricted to fixed-frequency-automatic measurements, the system is seen to be easily generalized to step-frequency measurements as well. The latter can be used to record automatically the complex impedance spectrum of an unknown microwave load when the frequency is changed. Component imperfections that may affect the system accuracy and comparison of the present system with other automatic measuring systems are discussed.**

I. Introduction

MULTIPLE-FIXED probes mounted on a waveguide or a transmission line have been used by many investigators to measure the complex impedance $\bar{Z}$ of an unknown microwave load. A previous article published in this TRANSACTIONS [1] has summarized some of the background work in the field. Also in the article quoted, a novel approach that clarifies certain design confusions and that allows new designs to be reached was reported in detail. Following this new approach, static measurements (measurements of the outputs of the three probes with hand calculation to predict the unknown $\bar{Z}$ at different frequencies) and discussions of possible effects of system performance due to component imperfections were reported in several papers [2]–[4]. These static measurements verify, to a certain degree of accuracy, the theory reported in the quoted article. The theory is also verified by other investigators using digital means [5].

In the present article, the reader will see a simplified, but improved, design. Based on this new design, a dc electronic signal processor was built and tested. The detail of the design, the calibration, and the experimental result of this new approach will be discussed in order, following a brief review of the fundamental theory.

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II. Theory

As reported in the quoted article [1], three fixed probes mounted on a lossless waveguide or a transmission line terminated by an unknown impedance $\bar{Z}_L$ (Fig. 1) can be used to measure automatically the phase and the magnitude of the complex $\bar{Z}_L$ at any fixed microwave frequency. The conditions that this measurement is made possible are

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